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The 'balanced force' concept for instrumentation of curved canals revisited

T. J. CHARLES & J. E. CHARLES
Laboratoire de Stomatologie, Université libre de Bruxelles / Anderlecht, Brussels, Belgium

Summary

The principal characteristics of K-type files are described and the succession of operations as prescribed by the 'balanced force' method of preparation of root canals, particularly curved root canals, examined. On the basis of a simple mathematical model, the motions of a file in a straight canal and in a curved canal are successively considered, and the merits of the recommended method highlighted and confirmed.

Keywords: **balanced force, root canal preparation.**

Introduction

Roane *et al.* (1985) advocated an original preparation method for curved root canals which allows the same degree of quality of the enlarging process as is usually required and obtained for straight canals, while avoiding ledge formation, transportation of the foramen or perforation. The method is known as the 'balanced force' technique. According to the authors, it is the outcome of extensive experimentation. This paper is not devoid of a certain empiricism, but the day-to-day utilization of this method has convinced the practitioner of its validity and merits. The aim of this study is to explain scientifically the balanced force technique, using a simplified mathematical model.

The manufacture and description of K-files

The manufacture of a K-file can proceed along the following lines. A rod of stainless steel, in the shape of a truncated pyramid with a square or triangular cross section, is twisted to obtain helicoidal cutting edges of determined pitch. The piece is then quenched and hardened in order to confer to the steel a high elastic limit and a sufficient tenacity. According to the cross-sectional shape a so called square or triangular file is obtained, i.e. a file with respectively four or three threads per pitch.

The circumscribed surface is a truncated cone but, as the conicity is mostly small, it will be disregarded and, for present purposes, the circumscribed surface will be regarded as a circular cylinder.

Cross section and cutting edges

The cross sections are squares or equilateral triangles that can be obtained from one another by rotations through the appropriate angle about the axis of the circumscribed cylinder (Fig. 1).

The cutting edges of the file are right-hand circular helices of radius (OA on Fig. 1)

$$R = \frac{c}{\sqrt{3}}$$

where c is the side of the equilateral triangle or

$$R = \frac{c}{\sqrt{2}}$$

where c is now the side of the square.

The origins of the helices are

$$\sigma = 0, \frac{2\pi R}{3}, \frac{4\pi R}{3}$$

for a triangular file
and

$$\sigma = 0, \frac{\pi R}{2}, \frac{3\pi R}{2}$$

for a square file.

Parametric equations of one of the cutting edges are, $\varphi = \sigma/R$ being the polar angle and σ the length of the arc AM' on the base circle, counted anticlockwise:

Correspondence: T. J. Charles, Laboratoire de stomatologie – CP 621, Université libre de Bruxelles / Anderlecht, Route de Lennik, 808, B-1070 Bruxelles, Belgium.

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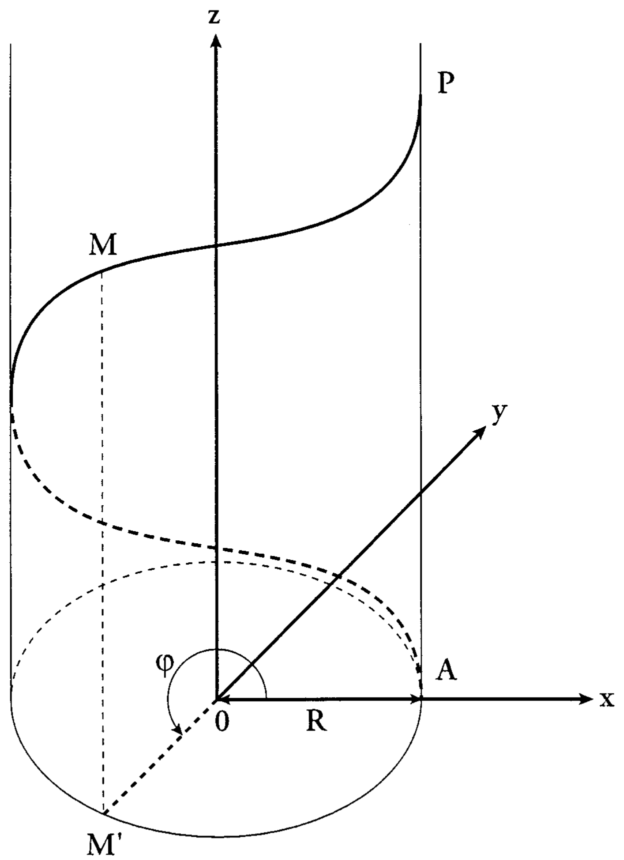


Fig. 1 Right-hand circular helix.

$$x = R\cos\frac{\sigma}{R}$$
$$y = R\sin\frac{\sigma}{R} \qquad \sigma \geqslant 0, k > 0 \qquad (1)$$
$$z = k\,\sigma$$

$2\pi kR$ is the pitch of the helix, i.e. the height gained by a point M describing the helix when its projection M' on the base circle has made one anticlockwise revolution; so, if M is taken in A, the pitch is AP (Fig. 1); k is called the parameter of the helix.

The radius of the inscribed coaxial cylinder is $\frac{R}{2}$ for a triangular file and $\frac{R}{\sqrt{2}}$ for a square file.

The inscribed cylinder is called the core of the file.

Meridian sections

The meridian section through point A of Fig. 1 is shown on Figs. 2, 3 for a triangular file. The different parts of the section can be deduced from one another by translations parallel to Oz. It can be shown that the angle β of the tangents in the points of the cutting edges is given by

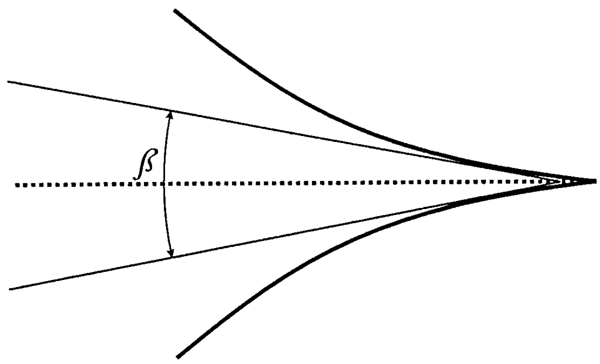


Fig. 2 Meridian section of the file.

$$\text{cotan}\frac{\beta}{2} = \frac{\sqrt{3}}{k}.$$

Utilization of a K-file for canal preparation

The 'balanced force' technique

This technique includes the following steps:

1 Placement into the canal and engagement of the file in the dentine by clockwise rotation and light inward pressure
2 Cutting and crushing of the dentine by anticlockwise rotation and inward pressure matched to the applied torque in such a way that no inward or outward motion of the file results
3 Repetition of the preceding operation after the file has been placed more deeply into the canal
4 At the appropriate time, removal of the debris by clockwise rotation with outward pulling and cleaning with irrigant.

The superiority of this technique in comparison with the more common procedure of filing by successive inward and outward motions lies in its ability to enlarge curved canals to the required final diameter without transportation of the foramen, ledge formation or perforation.

Cutting area

The dentine area of the cross section which can be crushed by the operation appears hatched in Fig. 4. It is a portion of a circular crown, cross section of the solid figure limited by the circumscribed cylinder and the canal wall.

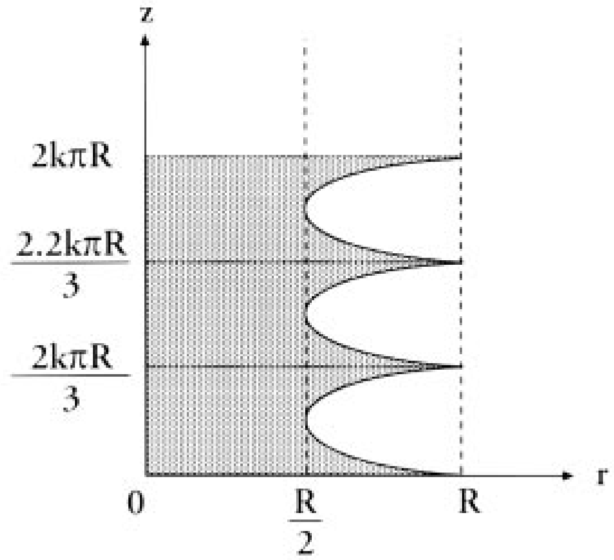


Fig. 3 Meridian section of a triangular K-type file.

Moments of inertia

If the elastic limit of the steel of the file is sufficiently high, the file will be able to progress in a curved canal without permanent deformation and to operate in rotative bending by clockwise as well as by anticlockwise rotation. According to the theory of elasticity, the restoring forces developed by the stressed instrument will be lesser if the moment of inertia (I) of the cross section is smaller. (To be precise, I is the moment of inertia with respect to an axis through the centre of the cross section and perpendicular to the bending plane.)

The moment of inertia of an equilateral triangle, a square or any regular polygon with respect to an axis of their plane through their centre is independent of the direction of that axis. Consequently, when the file, engaged in a curved canal, rotates about its longitudinal axis, the contact pressure between its cutting edges and the dentine, which is proportional to El (where E is the modulus of longitudinal elasticity of the steel), remains unchanged.

On the other hand, it is a classical result that the moments of inertia of an equilateral triangle and a square are respectively:

$$I_3 = \frac{3R^4\sqrt{3}}{32} \cong 0{,}16R^4 \text{ and } I_4 = \frac{R^4}{3} \cong 0{,}33R^4,$$

where R is the radius of the circumscribed circle (Fig. 4). This means that the triangular file is more flexible than a square file of the same diameter (and the same steel), that it will, for that reason, apply smaller restoring forces to the walls of a curved canal and therefore be less likely to cause irreversible damage during preparation. Triangular files should thus be preferred.

Motions of a file in a root canal

Enlargement of a straight canal does not pose any particular problems. It could be done by a succession of inward and outward motions accompanied or not by rotary motions. However, curved canals are different where departure from the balanced force technique can result in damage of the canal wall. As it is impossible to ascertain on the basis of a radiograph whether there is curvature in a bucco-lingual plane or not, it is expedient to apply the 'balanced force' technique as a general rule for the preparation of all root canals.

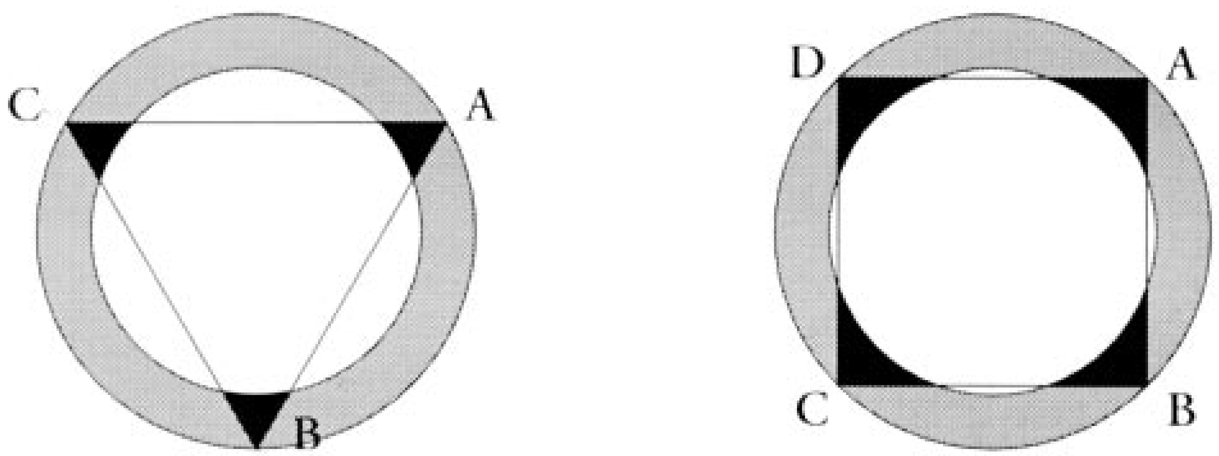


Fig. 4 Cutting area of a K-type file.

Motions in a straight canal

Before the more difficult problem of the curved canal is addressed, the preparation of the straight canals will be reviewed initially.

The file is supposed to be engaged in the dentine. The mechanical effects of the pulp are disregarded. The only possible motions of the file are a rotation about its axis (Oz), which coincides with the canal axis, and a translation along Oz. (Fig. 1)

The coordinates which define the position of the file are: Z, coordinate of the tip and φ, polar angle counted anticlockwise from Ox in the plane Oxy perpendicular to Oz.

A very general case will be considered where the applied forces are: an axial force

$$\overline{F} = F\overline{u}_z,\ F \gtrless 0$$

and forces tangential to the helve and equivalent to a torque

$\overline{C} = C\overline{u}_z,\ C \gtrless 0$. For the sake of convenience but without loss of generality, F and C are supposed to be constant on $\{t \geqslant 0\}$ In a problem of classical mechanics the intention would be to determine Z(t) and φ(t) on $\{t \geqslant 0\}$, the file being initially at rest. Here the kind of motion that will be started in $t = 0+$, i.e. the velocities $\dot{Z}$ and $\dot{\varphi}$ in the first instant will be determined.

The kinetic energy of the file in a vacuum would read

$$\left(\frac{1}{2}m\dot{Z}^2 + J\dot{\varphi}^2\right)$$

where m is its mass and J its moment of inertia with respect to the axis. For the same file in an environment which can develop reactions (for example dentine) there appears in the expression of the kinetic energy, in addition to the translation term $\left(\frac{1}{2}m\dot{Z}^2\right)$ and the rotation term $(J\dot{\varphi}^2)$ a rectangular term $(\dot{Z}\dot{\varphi})$, which accounts for the coupling of translation and rotation motions as is described by the third equation of (1). The kinetic energy of the right hand file is then a positive definite bilinear form:

$$T = \frac{1}{2}\left(a\dot{Z}^2 - 2b\dot{Z}\dot{\varphi} + c\dot{\varphi}^2\right)$$

with $a \geqslant 0,\ b \geqslant 0,\ c \geqslant 0,\ ac - b^2 > 0$. (2)

Media like pulp or crushed dentine which cannot develop reactions are assimilitated to vacuum; in those media the kinetic energy is therefore given by (2) with $b = 0$.

The potential energy of the file remains constant in the course of the operations. The Lagrange equations of the problem can be written as follows:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Z}}\right) - \frac{\partial T}{\partial Z} + \frac{\partial U}{\partial Z} = F \qquad Z(0) = Z_0,\ \dot{Z}(0) = 0,\quad t \geqslant 0 \qquad (3)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}}\right) - \frac{\partial T}{\partial \varphi} + \frac{\partial U}{\partial \varphi} = C \qquad \varphi(0) = \varphi_0,\ \dot{\varphi}(0) = 0,$$

With $\frac{\partial T}{\partial \dot{Z}} = a\dot{Z} - b\dot{\varphi},\ \frac{\partial T}{\partial Z} = 0,\ \frac{\partial U}{\partial Z} = 0,\ \frac{\partial T}{\dot{\varphi}} = c\dot{\varphi} - b\dot{Z},$

$\frac{\partial T}{\partial \varphi} = 0,\ \frac{\partial U}{\partial \varphi} = 0,$

(3) is rewritten

$$\frac{d}{dt}(a\dot{Z} - b\dot{\varphi}) = F \qquad (4)$$

$$\frac{d}{dt}(-b\dot{Z} + c\dot{\varphi}) = C$$

Making use of the initial conditions and integrating (4) leads to

$$a\dot{Z} - b\dot{\varphi} = Ft$$
$$-b\dot{Z} + c\dot{\varphi} = Ct$$

from which follows

$$\dot{Z} = \frac{cF + bC}{ac - b^2}t,\ \dot{\varphi} = \frac{bF + aC}{ac - b^2}t \qquad (5)$$

Some particular cases:

$C > 0,\ F = 0.$

This means that a clockwise torque is applied without axial force, in other words, screwing motion without pushing or pulling.

It follows from (5)

$$\dot{Z} = \frac{bCt}{ac - b^2} > 0,\ \dot{\varphi} = \frac{aCt}{ac - b^2} > 0,\ \ddot{\varphi} = \frac{aC}{ac - b^2} > 0 \qquad (6)$$

The last relation entails that a tangential force is applied to the dentine; if its breaking point is reached (i.e. if C is sufficient) the instrument cuts the dentine and moves inwards into the canal ($Z > 0$).

$C < 0,\ F > 0$ and such that $cF + bC = 0$ (7)

This is the situation advocated by Roane *et al* (1985) : an anticlockwise torque is applied, (an unscrewing motion) accompanied by an axial force towards the apex (a push) matched to the torque in such a way that relation (7) is satisfied: (7) is the mathematical expression and key condition of the so called 'balanced force' technique.

It follows from (5):

$$\dot{Z} = 0\,,\ \dot{\varphi} = \frac{aC + bF}{ac - b^2}t = \frac{C}{c}t < 0 \qquad (8)$$

$$\ddot{Z} = 0\,,\ \ddot{\varphi} = \frac{C}{c} < 0$$

It becomes apparent that, if (C) is sufficiently increased (with concomitant increase of F in such a way that (7) is satisfied), the tangential force that, in view of the last relation of (8), is applied to the dentine, eventually reaches the breaking point thereof; the instrument then starts an anticlockwise rotation, cutting the dentine without any inward or outward motion in the canal ($\dot{Z} = 0$).

The practitioner should have no difficulty in satisfying and maintaining condition (7), he or she only need adjust the inward pressure to prevent the absence of translatory motion of the file.

Prevention of debris accumulation. When enlargement of the canal is accomplished, it can be observed that a clockwise rotation of the file results in no axial translatory motion and elevates the debris toward the crown.

The material surrounding the file is now made up of loose pulp and dentine debris unable to develop reactions.

The fact is mathematically rendered by the nullity of coefficient b in (2). It follows now from (5) where b and F are made equal to 0

$$\dot{Z} = 0 \text{ and } \dot{\varphi} = \frac{C}{c}t < 0 \qquad (9)$$

This means that a clockwise rotation without concomitant translation is started; the file works then as an Archimedean screw, loads the debris into the flutes and elevates them away from the apex. The operation can be accelerated without any drawback by combining the clockwise rotation with an outward movement. However debris are removed mainly by the use of a copious irrigation of the root canal with a NaOCl solution.

Motions in a curved canal

Hypotheses and solution. The file, guided by the canal wall and engaged in the dentine is supposed to have undergone a plane bending in the plane Oxz. The variable β is the angle formed by Oz and the tangent in a variable point of the canal axis. Parametric equations of that axis are:

$$x = g(\beta) \qquad 0 \leqslant \beta < \frac{\pi}{2}$$

$$y = 0 \qquad 0 \leqslant x$$

$$z = f(\beta) \qquad 0 \leqslant z$$

with f (0) = (0), g(O) = 0, f and g monotonously increasing functions (Fig. 5). The values of β and z at the tip of the file are respectively denoted γ and Z; from $Z = f(\gamma)$ follows $\gamma = f^{-1}(Z) = h(Z)$.

The torsion of the instrument, which could be neglected in the case of a straight canal, has now to be taken into account. As a consequence, the polar angle φ, counted as usual anticlockwise from a axis parallel to Ox, varies along the file. If φ is assumed to be equal to, say, zero in the tip cross section at the instant that the rotation is going to take place, it will be at the same instant, different from zero in the base cross section ($\varphi_0 \neq 0$).

The kinetic energy of the file is still given by (2) where $\dot{\varphi}$ is to be replaced by its value in the tip cross section and where the coefficients a, b, c, though still satisfying the inequalities of (2), now have other values than in (2); the remark about coefficient b, immediately following (2) remains valid. The potential energy includes two terms, a bending term and a torsion term:

the bending energy is $U_b = \frac{EI}{l}\gamma$,

where E and I have been defined previously and where *l* is the length of the file (which will be in the sequel assimilated to Z).

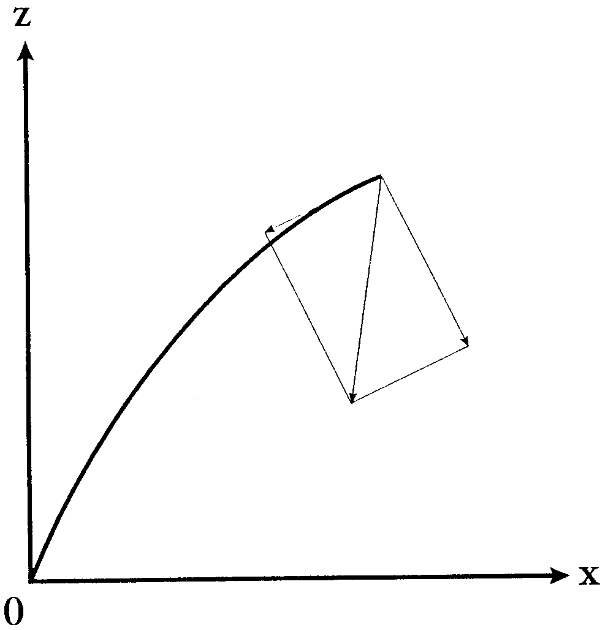
The torsion energy is $U_t = \frac{GI_p}{l}\varphi \operatorname{sgn} \varphi$

where G is the modulus of transverse elasticity, where I_p is the polar moment of inertia of the cross section and where sgnφ is 1 for $\varphi > 0$ and -1 for $\varphi < 0$ where φ is to be replaced by its value in the base cross section and *1* by Z.

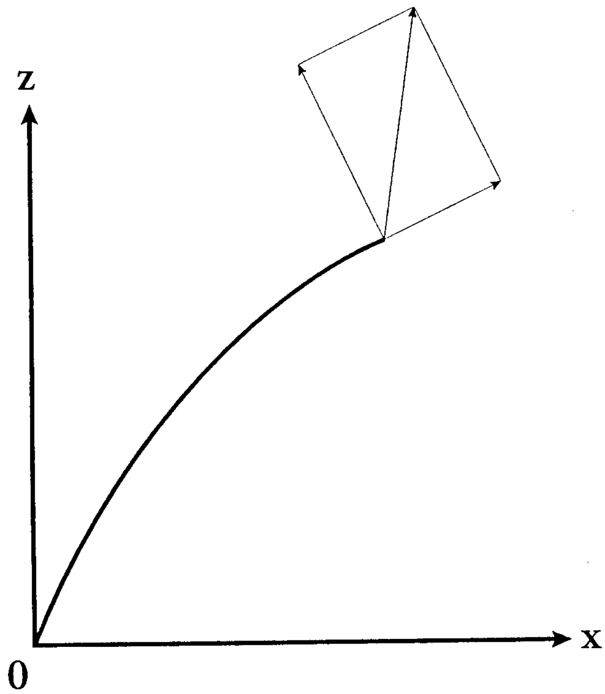
The potential energy can thus be written

$$U = \frac{EI}{Z}h(Z) + \frac{GI_p}{l}\varphi \operatorname{sgn}\varphi$$

The Lagrange equations are the same as in (3) but with, now,



case (15)



case (16)

Fig. 5 Forces at the tip of a K-type file in a curved canal by a pull (case 15) and a push (case 16) motion.

$$\frac{\partial T}{\partial \dot{Z}} = a\dot{Z} - b\dot{\varphi}, \frac{\partial T}{\partial Z} = 0, \frac{\partial T}{\partial \varphi} = b\dot{Z} + c\dot{\varphi}, \frac{\partial T}{\partial \varphi} = 0,$$

$$\frac{\partial U}{\partial Z} = EI\left(\frac{h'(Z)}{Z} - \frac{h(Z)}{Z^2}\right) - \frac{GI_P}{Z^2}\varphi \text{ sgn } \varphi, \frac{\partial U}{\partial \varphi} = \frac{GI_P}{Z}\text{sgn}\varphi,$$

they have to be rewritten as follows:

$$\frac{d}{dt}(a\dot{Z} - b\dot{\varphi}) + EI\left(\frac{h'(Z)}{Z} - \frac{h(Z)}{Z^2}\right) - \frac{GI_P}{Z^2}\varphi\text{sgn}\varphi = F$$
$$\frac{d}{dt}(-b\dot{Z} + c\dot{\varphi}) + \frac{GI_P}{Z}\text{sgn}\varphi = C. \quad (10)$$

(10) is integrated by the iteration method of Picard limited to the first iteration. With the initial conditions as written in (3) this leads to

$$a\dot{Z} - b\dot{\varphi} \cong \int_0^t F dt - \int_0^t \left[EI\left(\frac{h'(Z_0)}{Z_0} - \frac{h(Z_0)}{Z_0^2}\right) - \frac{GI_P}{{Z_0}^2}\varphi_0\text{sgn}\varphi_0\right]dt$$
$$-b\dot{Z} + c\dot{\varphi} \cong \int_0^t C dt - \int_0^t \frac{GI_P}{{Z^2}_0}\text{sgn}\varphi_0 dt.$$

Introducing for convenience the notations

$$f \equiv \frac{EI}{Z_0}\left[h'(Z_0) - \frac{h(Z_0)}{Z_0^2}\right] \text{ and } \tau \equiv \frac{GI_P}{Z_0} > 0$$

and solving (11) for $\dot{Z}$ and $\dot{\varphi}$ leads to the final result:

$$\dot{Z} = \frac{1}{ac - b^2}\left[c\left(F - f + \frac{\tau}{Z_0}\varphi_0\text{sgn}\varphi_0\right) + b(C - \tau\text{sgn}\varphi_0)\right]t$$
$$\dot{\varphi} = \frac{1}{ac - b^2}\left[b\left(F - f + \frac{\tau}{Z_0}\varphi_0\text{sgn}\varphi_0\right) + a(C - \tau\text{sgn}\varphi_0)\right]t \quad (12)$$

(12) is similar to (5) except for the terms f and τ sgn φ_0 which represent the effect of bending and torsion and affect respectively the axial force F and the torque C.

Some particular cases

$$\text{sgn } \varphi_0 = -1, C < -\tau, F > f + \frac{\tau}{Z_0}\varphi_0$$

$$\text{and such that } cF + bC = c(f + \frac{\tau}{Z_0}\varphi_0) - b\tau. \quad (13)$$

This is again the situation advocated by Roane *et al. (1985)*; (13) is similar to (7) except for the adequate corrections to C and F which result from torsion and bending. The presence of condition sgn $\varphi_0 = -1$ can be explained as follows : an 'unscrewing' torque (C<O) is applied on the handle of the file and its magnitude (C) progressively increased; initially the sole effect is a torsion of the instrument, φ at the tip ramaining unchanged ($\varphi = 0$) while the base cross section becomes more and more negative and is $\varphi_0 < 0$ when the breaking point of the dentine is reached, the motion starts and the matched axial force F towards the apex is applied.

It follows from (12) and (13):$\dot{Z} = 0$,

$$\dot{\varphi} = \frac{C + \tau}{c} t < 0 \, , \ddot{Z} = 0 \, , \ddot{\varphi} = \frac{C + t}{c} < 0 \tag{14}$$

With (13) and (14) replacing (7) and (8), the conclusions drawn in the case of the straight canal remain entirely valid.

$$F < f - \frac{\tau\varphi_0}{Z_0} \text{sgn } \varphi_0 \text{ and C such that } C - \tau\text{sgn}\varphi_0 = -\frac{b}{a}\left(F - f + \frac{\tau\varphi_0}{Z_0}\text{sgn}\varphi_0\right) \tag{15}$$

$$\text{or } F > f - \frac{\tau\varphi_0}{Z_0} \text{sgn } \varphi_0 \text{ and C such that } C - \tau\text{sgn}\varphi_0 = -\frac{b}{a}\left(F - f + \frac{\tau\varphi_0}{Z_0}\text{sgn } \varphi_0\right) \tag{16}$$

The practitioner simply exerts a traction on the file (case (15)) or a pressure toward the apex (case 16) while preventing the instrument from turning (*cf.* (17)). These are the two components of a push and pull motion used with the purpose of scraping the dentine.

It follows from (12) :

$$\dot{Z} = \frac{c\left(F - f + \frac{\tau\varphi_0}{Z_0}\text{sgn}\varphi_0\right)}{ac - b^2} t \begin{cases} < 0 \text{ in case } (15) \\ > 0 \text{ in case } (16) \end{cases}, \dot{\varphi} = 0 \tag{17}$$

$$\ddot{Z} = \frac{c\left(F - f + \frac{\tau\varphi_0}{Z_0}\text{sgn}\varphi_0\right)}{ac - b^2} t \begin{cases} < 0 \text{ in case } (15) \\ > 0 \text{ in case } (16) \end{cases}, \ddot{\varphi} = 0 \tag{18}$$

It appears from (18) that a force parallel to Oz and directed toward the crown (case 15) or the apex (case 16) is applied to the tip of the file. This force can be decomposed into a component tangential to the file (and responsible for its moving to the crown or the apex while scraping the dentine) and a component normal to the file and directed toward its convex side (case 15) or its concave side (case 16). If (F) is sufficiently increased in order to crush the dentine, the latter component reaches the breaking point of the dentine and can cause ledge formation, transportation of the foramen and local straightening of the canal (Fig. 5).

References

ROANE JB, SABALA CL, DUNCANSON MG Jr (1985) The 'Balanced Force' concept for instrumentation of curved canals. *Journal of Endodontics* **11**, 203–211.